Ex (1) art 146 Byerly Int Calculus

$x^2+y^2+z^2=a^2$ Equation of Surface of Sphere

$x^2-ax+y^2=0$ — — Cylinder

$$\sigma = 8\int_0^a\int_0^{\sqrt{ax-x^2}} \frac{\sqrt{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2}}{\frac{\partial u}{\partial z}}\, dy\, dx$$

$$\sigma = 8\int_0^a\int_0^{\sqrt{ax-x^2}} \frac{a\, dy\, dx}{z} = 8\int_0^a\int_0^{\sqrt{ax-x^2}} \frac{a\, dy\, dx}{\sqrt{a^2-x^2-y^2}}$$

$$= 8a\int_0^a \sin^{-1}\frac{\sqrt{ax-x^2}}{\sqrt{a^2-x^2}}\, dx = 8a\int_0^a \sin^{-1}\sqrt{\frac{x}{a+x}}\, dx$$

Let $u = \sin^{-1}\sqrt{\frac{x}{a+x}}$ $dv = dx$

$$\int \sin^{-1}\sqrt{\frac{x}{a+x}}\, dx = x\sin^{-1}\sqrt{\frac{x}{a+x}} - \frac{\sqrt{a}}{2}\int\frac{\sqrt{x}}{a+x}\, dx$$

$$\int_0^a \sin^{-1}\sqrt{\frac{x}{a+x}}\, dx = a\sin^{-1}\frac{\sqrt{2}}{2} - \frac{\sqrt{a}}{2}\int\frac{\sqrt{x}}{a+x}\, dx$$

Let $w=\sqrt{x}$
$w^2 = x$
$2w\, dw = dx$

$$\int\frac{\sqrt{x}\, dx}{a+x} = 2\int\frac{w^2\, dw}{a+w^2} = 2\left(w - \sqrt{a}\tan^{-1}\frac{w}{\sqrt{a}}\right)$$

$$\therefore \sigma = 8a\int_0^a \sin^{-1}\sqrt{\frac{x}{a+x}}\, dx = 8a\left(a\sin^{-1}\frac{\sqrt{2}}{2} - \sqrt{a}\left(w - \sqrt{a}\tan^{-1}\frac{w}{\sqrt{a}}\right)\right)$$

$$\sigma = 8a\left(a\sin^{-1}\frac{\sqrt{2}}{2} - \sqrt{a}\left(\sqrt{a} - \sqrt{a}\tan^{-1}1\right)\right)$$

$$= 8a\left(a\sin^{-1}\frac{\sqrt{2}}{2} - a + a\frac{\pi}{4}\right)$$

$$= 8a\left(a\frac{\pi}{4} + a\frac{\pi}{4} - a\right)$$

$$= 8a\left(a\frac{\pi}{2} - a\right) = 4\pi a^2 - 8a$$ (as it ought to be)

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To find the equation of the surface of a cone whose vertex is at the origin & whose axis coincides with the axis of z and whose base parallel to the plane xy is $x^{2/3} + y^{2/3} = a^{2/3}$

Since the origin is on the surface, there will be no constant term.

Since when $z = c$ the section becomes $x^{\frac{2}{3}} + y^{2/3} = a^{2/3}$

$\frac{z}{c}$ must be a factor in the second member, & the equation must be of the form $x^{2/3} + y^{2/3} = a^{2/3}\left(\frac{z}{c}\right)^n$

Since the section of the cone by the plane xz is two straight lines, when $y = 0$ $x^{\frac{2}{3}} = a^{\frac{2}{3}}\left(\frac{z}{c}\right)^n$ must be straight lines & ∴ $\frac{z}{c}$ & x must enter to the same degree

That is $x^{2/3} = a^{\frac{2}{3}}\left(\frac{z}{c}\right)^{\frac{2}{3}} = \left(\frac{az}{c}\right)^{\frac{2}{3}}$

And the equation of the surface becomes $x^{\frac{2}{3}} + y^{\frac{2}{3}} = \left(\frac{az}{c}\right)^{\frac{2}{3}}$

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$$u = x^{\frac{2}{3}} + y^{\frac{2}{3}} - \frac{a^{\frac{2}{3}}}{c^{2/3}} z^{2/3} = 0$$ ②

$$\frac{du}{\partial x} = \frac{2}{3} x^{-\frac{1}{3}}, \quad \frac{\partial u}{\partial y} = \frac{2}{3} y^{-\frac{1}{3}}, \quad \frac{\partial u}{\partial z} = -\frac{2}{3} \frac{a^{2/3}}{c^{2/3}} z^{-\frac{1}{3}}$$

$$\left(\frac{\partial u}{\partial x}\right)^2 = \frac{4}{9} x^{-\frac{2}{3}}, \quad \left(\frac{\partial u}{\partial y}\right)^2 = \frac{4}{9} y^{-\frac{2}{3}}, \quad \left(\frac{\partial u}{\partial z}\right)^2 = \frac{4}{9} \frac{a^{\frac{4}{3}}}{c^{4/3}} z^{-\frac{2}{3}}$$

$$\frac{\sqrt{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2}}{\frac{\partial u}{\partial z}} = \sqrt{\frac{\frac{4}{9} x^{-\frac{2}{3}} + \frac{4}{9} y^{-\frac{2}{3}} + \frac{4}{9} \frac{a^{4/3}}{c^{4/3}} z^{-\frac{2}{3}}}{\frac{4}{9} \frac{a^{4/3}}{c^{4/3}} z^{-\frac{2}{3}}}} = \sqrt{1 + \left(x^{-\frac{2}{3}} + y^{-\frac{2}{3}}\right) \frac{c^{4/3}}{a^{4/3} z^{-\frac{2}{3}}}}$$

$$= \sqrt{1 + \left(\frac{1}{x^{2/3}} + \frac{1}{y^{2/3}}\right) \frac{c^{4/3} z^{2/3}}{a^{4/3}}} = \sqrt{1 + \frac{x^{2/3} + y^{2/3}}{x^{2/3} y^{2/3}} \frac{c^2}{a^2} \left(x^{2/3} + y^{2/3}\right)}$$

$$= \sqrt{1 + \frac{c^2}{a^2} \frac{\left(x^{2/3} + y^{2/3}\right)^2}{x^{2/3} y^{2/3}}}$$

$$\frac{a^{2/3}}{c^{2/3}} z^{2/3} = x^{\frac{2}{3}} + y^{\frac{2}{3}}$$

$$z^{\frac{2}{3}} = \frac{c^{2/3}}{a^{2/3}} \left(x^{\frac{2}{3}} + y^{2/3}\right)$$

$$\frac{c^{4/3}}{a^{4/3}} z^{2/3} = \frac{c^2}{a^2} \left(x^{2/3} + y^{2/3}\right)$$

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Equation (3) is

$$\sigma = \frac{4}{a}\iint x^{-\frac{1}{3}} y^{-\frac{1}{3}} \sqrt{a^2 x^{2/3} y^{2/3} + c^2\left(x^{2/3}+y^{2/3}\right)^2}\, dy\, dx \qquad (3)$$

where the base is $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{2/3}$

Now if the base be $x'^2 + y'^2 = a^{2/3}$

σ may be found by substituting x'^2 for $x^{\frac{2}{3}}$ & y'^2 for $y^{2/3}$

$2x'dx' = \frac{2}{3}x^{-\frac{1}{3}}dx$

or $x'dx' = \frac{1}{3}x^{-\frac{1}{3}}dx$

or $3x'dx' = x^{-\frac{1}{3}}dx$

And similarly

$3y'dy' = y^{-\frac{1}{3}}dy$

& (3) becomes

$$\sigma = \frac{36}{a}\iint x'y'\sqrt{a^2x'^2y'^2 + c^2\left(x'^2+y'^2\right)^2}\, dy'\, dx'$$

or removing accents

$$\sigma = \frac{36}{a}\iint xy\sqrt{a^2x^2y^2 + c^2\left(x^2+y^2\right)^2}\, dy\, dx \qquad (4)$$

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$$\sigma = 6a\int_0^{\frac{\pi}{2}} \sin\varphi\cos\varphi\sqrt{a^2\sin^2\varphi\cos^2\varphi + c^2}\,d\varphi$$

$u = \sin^2\varphi$ when $\varphi = 0$ $u = 0$, when $\varphi = \frac{\pi}{2}$, $u = 1$

$$du = 2\sin\varphi\cos\varphi\,d\varphi$$

$$\sigma = 3a\int_0^1\sqrt{a^2u(1-u)+c^2}\,du$$

$$= 3a^2\int_0^1\left(u-u^2+\frac{c^2}{a^2}\right)^{\frac{1}{2}}du$$

$$= 3a^2\int_0^1\left(u-u^2+p^2\right)^{\frac{1}{2}}du$$

Let $\sqrt{u-u^2+p^2} = p + zu$

$$u - u^2 + p^2 = p^2 + 2pzu + z^2u^2$$

$$u - u^2 = 2pzu + z^2u^2$$

$$1 - u = 2pz + z^2u$$

$$u = \frac{1-2pz}{1+z^2}$$

$$du = \frac{-2p+2pz^2-2z}{(1+z^2)^2}\,dz$$

$$= -2\,\frac{p-pz^2+z}{(1+z^2)^2}\,dz$$

$$\sqrt{u-u^2+p^2} = p + z\,\frac{1-2pz}{1+z^2}$$

$$= \frac{p+pz^2+z-2pz^2}{1+z^2}$$

$$= \frac{p-pz^2+z}{1+z^2}$$

$$\therefore \sqrt{u-u^2+p^2}\,du = \frac{-2(p-pz^2+z)^2}{(1+z^2)^3}\,dz$$

$$\sigma = 3a^2\int -2\frac{(p-pz^2+z)^2}{(1+z^2)^3}\,dz$$

$$\sigma = -6a^2\int\frac{(p-z+pz^2)^2}{(1+z^2)^3}\,dz$$

Expand & integrate each term using reduction formulae.

The limits of u are 0 and 1
The — " — z are $\frac{1}{2p}$ and 0

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