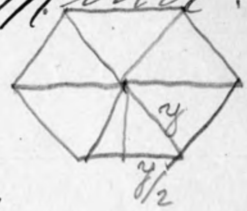
Miller.



V

y = radius hexagon

" = 1 side " $\frac{y}{2} = \frac{1}{2}$ side.

Altitude triangle $= \sqrt{y^2 - \frac{y^2}{4}} = \frac{\sqrt{3}}{2}y.$

$$\frac{\sqrt{3}}{2}y \cdot \frac{y}{2} = \frac{\sqrt{3}}{4}y^2 = \text{area 1 triangle}$$

$$\frac{3}{2}\sqrt{3}\,y^2 = \text{6 faces} \qquad x^2+y^2=a^2.$$

$$V = \frac{3}{2}\sqrt{3}\int_0^a (a^2-x^2)\,dx = \frac{3}{2}\sqrt{3}\left[a^2x - \frac{x^3}{3}\right]_0^a = \sqrt{3}a^3$$

$$\sqrt{3}a^3 \cdot 2 = 2\sqrt{3}a^3.$$

$$r = a(\sec\varphi - \tan\varphi)$$

$$r^2 = a^2(\sec^2\varphi - 2\sec\varphi\tan\varphi + \tan^2\varphi) \qquad 13$$

$$\frac{1}{2}\int r^2\,d\varphi = \frac{a^2}{2}\int(\sec^2\varphi - 2\sec\varphi\tan\varphi + \sec^2\varphi - 1)\,d\varphi$$

From $\varphi = 0$ to $\varphi = \frac{\pi}{2}$ for half loop

$$\frac{1}{2}A = \frac{a^2}{2}\left(\tan\varphi - 2\sec\varphi + \tan\varphi - \varphi\right)\Big]_0^{\frac{\pi}{2}} + C$$

$A = 0$ when $\varphi = 0$ ∴ $C = a^2$

$$\frac{1}{2}A = \frac{a^2}{2}(2\tan\varphi - 2\sec\varphi - \varphi) + a^2$$

~~when $\varphi = \frac{\pi}{2}$ $\frac{1}{2}A = \frac{a^2}{2} - \frac{\pi a^2}{4} + a^2$~~

~~$A = \frac{1}{2}\left(2a^2 - \frac{\pi}{2}\right)$~~ $= a^2 - \frac{\pi}{4}a^2 = a^2\left(1 - \frac{\pi}{4}\right)$

$$\frac{1}{2}A = \frac{a^2}{2}\left(-\frac{\pi}{2}\right) + a^2$$

$$A = a^2\left(-\frac{\pi}{2}\right) + 2a^2$$

$$= 2a^2\left(1 - \frac{\pi}{4}\right)$$

$$x\,dy = \left[(a+b)\cos\theta - b\cos\frac{a+b}{b}\theta\right]\left[(a+b)\left(\cos\theta - \cos\frac{a+b}{b}\theta\right)\right]d\theta$$

$$= \Big[(a+b)^2\cos^2\theta - b(a+b)\cos\theta\cos\frac{a+b}{b}\theta - (a+b)^2\cos\theta\cos\frac{a+b}{b}\theta$$
$$+ b(a+b)\cos^2\frac{a+b}{b}\theta\Big]d\theta$$

$$y\,dx = \left[(a+b)\sin\theta - b\sin\frac{a+b}{b}\theta\right]\left[(a+b)\left(-\sin\theta + \sin\frac{a+b}{b}\theta\right)\right]d\theta$$

$$= \Big[(a+b)^2(-\sin^2\theta) + b(a+b)\sin\theta\sin\frac{a+b}{b}\theta - b(a+b)\sin^2\frac{a+b}{b}\theta\Big]d\theta$$
$$+ (a+b)^2\sin\theta\sin\frac{a+b}{b}\theta$$

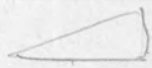
$$x\,dy - y\,dx = (a+b)^2 - b(a+b)\left(\cos\theta\cos\frac{a+b}{b}\theta + \sin\theta\sin\frac{a+b}{b}\theta\right)$$
$$- (a+b)^2\left(\cos\theta\cos\frac{a+b}{b}\theta + \sin\theta\sin\frac{a+b}{b}\theta\right) + b(a+b)$$

$$= \left[(a+b)(a+2b) - b(a+b)\cos\frac{a\theta}{b} - (a+b)^2\cos\frac{a\theta}{b}\right]d\theta$$

$$= \left[(a+b)(a+2b) - (a+b)(a+2b)\cos\frac{a\theta}{b}\right]d\theta$$

$$= (a+b)(a+2b)\left[1 - \cos\frac{a\theta}{b}\right]d\theta$$

$$\int\rho^2\,d\varphi = (a+b)(a+2b)\int\left(1 - \cos\frac{a\theta}{b}\right)d\theta$$



$$\int\rho^2\,d\theta = (a+b)(a+2b)\left(\theta - \frac{b}{a}\sin\frac{a\theta}{b}\right)\Bigg|_0^{\frac{2\pi b}{a}}$$

$$= (a+b)(a+2b)\left(\frac{2\pi b}{a} - \frac{b}{a}\sin 2\pi b\right)$$

$$= (a+b)(a+2b)\frac{2\pi b}{a}$$

$\frac{1}{2}\rho^2\theta$

$\frac{1}{2}a(2\pi b)$

πab

$$\frac{1}{2}\int\rho^2\,d\theta = (a+b)(a+2b)\pi\frac{b}{a} - a\pi b$$

$$= \pi b\left[\frac{(a+b)(a+2b)}{a} - a\right]$$

$$= \frac{\pi b}{a}\left[(a+b)(a+2b) - a^2\right]$$

$$= \frac{\pi b}{a}\left[\cancel{a^2} + ab + 2ab + 2b^2 - \cancel{a^2}\right]$$

$$= \frac{\pi b}{a}(3ab + 2b^2) = \frac{b^2}{a}(3a + 2b)\pi$$