Mathematics II

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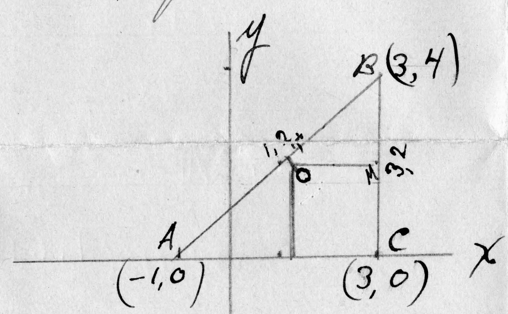
2: Find the equation of the circle which circumscribes the triangle whose sides are represented by the equations $y=0$ $y-x=1$ and $x=3$ Draw the figure.

$$\begin{array}{l} y=0 \\ y=x+1 \\ \hline x=-1 \\ y=0 \end{array} \qquad \begin{array}{l} x=3 \\ -x=-y+1 \\ \hline -y+3+1=0 \\ y=3+1 \text{ or } 4 \\ x=3 \end{array} \qquad \begin{array}{l} y=0 \\ x=3 \end{array} \quad \text{(Combining)}$$

Then with the above co-ordinates the triangle is constructed as follows.



Now having found the points of the triangle we will use the equation of a line passing through two given points to find the slope so that we can find the equation of the $\perp$ midway between BC, and in like manner AB.

Then by geometry the $\perp$ erected at the middle point of AC will intersect the $\perp$ erected at the middle of BC and AB, and there center will be the center of the circle.

Then proceeding we have, the equation of a line passing through two give points: take B and C first.

$\sin 22\tfrac{1}{2}^\circ : \sin 67\tfrac{1}{2}^\circ = \tfrac{5}{4} : OB$

$OB = \frac{5}{4}\frac{\sin 67\frac{1}{2}^\circ}{\sin 22\frac{1}{2}^\circ}$

$\frac{\sin 3x}{\sin x} = \frac{3\sin x - 4\sin^3 x}{\sin x}$

$= 3 - 4\sin^2 x$

& $\frac{\sin 67\frac{1}{2}^\circ}{\sin 22\frac{1}{2}^\circ} = 3 - 4\sin^2 22\tfrac{1}{2}^\circ$

$\therefore OB = \frac{5}{4}(3 - 4\sin^2 22\tfrac{1}{2}^\circ)$

But $4\sin^2 22\tfrac{1}{2}^\circ = 2(1 - \cos 45^\circ)$

$\therefore OB = \frac{5}{4}\left[3 - 2(1 - \cos 45^\circ)\right]$

$OB = \frac{5}{4}(1 + 2\cos 45^\circ)$

$\cos 45^\circ = \tfrac{1}{2}\sqrt{2}$

$OB = \frac{5}{4}(1 + \sqrt{2})$

$OG^2 = \overline{OB}^2 + \overline{GB}^2$

$\overline{OG}^2 = \frac{25}{16}(1+\sqrt{2})^2 + \left(\frac{5}{4}\right)^2$

$\overline{OG}^2 = \frac{25}{16}\left[1 + 2\sqrt{2} + 2 + 1\right]$

$OG^2 = \frac{25}{16}\left[4 + 2\sqrt{2}\right]$

$OG = \frac{5}{4}\sqrt{4 + 2\sqrt{2}}$

$OD = OG - \frac{5}{4}$

$\therefore OD = \frac{5}{4}\left[\sqrt{4+2\sqrt{2}} - 1\right]$

& $OE = \frac{5}{4}\left[\sqrt{4+2\sqrt{2}} + 1\right]$

Circumference of inner circle

$= 2\pi \cdot \frac{5}{4}\left[\sqrt{4+2\sqrt{2}} - 1\right]$

$= \frac{5}{2}\pi\left[\sqrt{4+2\sqrt{2}} - 1\right]$ (A)

Circumference of outer circle

$= \frac{5}{2}\pi\left[\sqrt{4+2\sqrt{2}} + 1\right]$ (B)

$\sqrt{2} = 1.416$ approximately

$2\sqrt{2} = 2.832$

$4 + 2\sqrt{2} = 6.832$

$\sqrt{4+2\sqrt{2}} = 2.612$

$\therefore$ (A) becomes

$\frac{5}{2}\pi(1.612)$

or $5\pi(0.806)$

or $\pi(4.03)$

or 12.660648 inches

$$\begin{array}{r} 3.1416 \\ 4.03 \\ \hline 94248 \\ 125664 \\ \hline 12660648 \end{array}$$

(B) becomes

$\frac{5}{2}\pi(3.612)$

or $5\pi(1.806)$

or $\pi(9.03)$

or 28.368648 inches

$$\begin{array}{r} 3.1416 \\ 9.03 \\ \hline 94248 \\ 282744 \\ \hline 28.368648 \end{array}$$

Or roughly – Circum of inner circle $12\frac{2}{3}$ inches

" " outer " $28\frac{1}{3}$ —